Hole Sensitivity Analysis for Topology Optimization in Electrostatic System Using Virtual Hole Concept and Shape Sensitivity

Kang Hyouk Lee, Seung Geon Hong, Myung Ki Baek, and Il Han Park

School of Electronic and Electrical Engineering, Sungkyunkwan University, Suwon 440-746 Korea, ihpark@skku.ac.kr

In this paper, the hole sensitivity formula for topology optimization in electrostatic system is derived using the continuum shape sensitivity and the virtual hole concept. The hole sensitivity gives additional information to guide the searching direction of the optimization to the global minimum. The hole sensitivity formula is the simple closed form. The shape and hole sensitivity provides the direction of topology variation and the variation is represented by the level set method with finite element method. The numerical example is tested and are compared to prove the usefulness of the hole sensitivity in electrostatic system.

***Index Terms*—Continuum shape sensitivity, Electrostatic system, Optimization, Topology sensitivity formula, Virtual hole**

I. Introduction

Numerical topology optimization in electromagnetic system has been one of the most important issues for engineers. The sensitivity analysis based on the material derivative concept of continuum mechanics was developed about two decades ago. The sensitivity provides the direction of the topology variation. In the continuum sensitivity analysis, the accurate sensitivity is calculated using the analytically derived sensitivity formula. To handle the topology variation numerically, the level set method was adopted. The feasibility of the optimization using the continuum sensitivity analysis and the level set method with finite element method have been demonstrated over the last decade [1], [2]. However, the possibility trapped in the local minimum still remains because topology tends to be gradually simpler as the optimization progresses with the level set method. Therefore, a numerical technique modifying topology during the optimization is necessary to guide the searching direction to the global minimum.

Recently, a new method of topology sensitivity concept including the hole sensitivity with the level set method was presented by the studies for structural optimization [3], [4]. In this method, the hole sensitivity gives the information about positions where the hole should exist in the material region. Then, the hole generation and the deformation of design variable are implemented by the level set method. The hole sensitivity method can relieve the drawback of the level set method explained above.

However, the hole sensitivity method cannot be directly used in topology optimization of electromagnetic system. In the structural optimization, the hole region is not the stress field analysis domain. On the other hand, the electromagnetic field exists in the hole region for electromagnetic optimization. That is, the hole surface in the structural case is treated as the outer boundary, but the one in electromagnetic case is the interface between two different materials. Thus, the hole sensitivity formula should be derived using the shape sensitivity on the interface.

In this paper, the hole sensitivity formula for electrostatic system is analytically derived in a closed form using the virtual hole concept and the continuum shape sensitivity. The hole sensitivity for magnetostatic system was already presented by the resent study, but not for electrostatic system [5]. The form of the hole sensitivity formulas for two systems is similar although the characteristics of the material properties and the state variables are quite different. Its usefulness is proved by a numerical test model as the conventional and the hole sensitivity methods are compared.

II. Hole Sensitivity in Electrostatic System

Fig. 1 shows an electrostatic system which consists of two dielectrics containing a hole. The design domain Ω is divided into the material interface $\gamma = \gamma_o + \gamma_h$. The subscripts 1 and 2, o, and h mean two materials, outer material interface, and hole material interface, respectively. The center of hole is at $\mathbf{x}$ and its radius is ρ. ε is the permittivity and $\mathbf{n}$ is the unit normal vector on the material interface. Assuming a perturbation by the hole generation makes γ_o invariant, the objective function F depends only on $\mathbf{x}$ and ρ. The shape sensitivity for F is the boundary integration over γ_h.

$$\dot{\mathrm{F}}(\mathbf{x}, \rho) = \int_{\gamma_h} (\varepsilon_1 - \varepsilon_2) \mathbf{E}(V_1) \cdot \mathbf{E}(\lambda_2) \mathrm{V_n} \, d\Gamma. \tag{1}$$

where, V is the electric potential which is the state variable, λ is the adjoint variable, and $\mathrm{V_n}$ is the normal component of the velocity vector on γ_h. As the hole is small enough in comparison with the surrounding material, electric field outside the hole $\mathbf{E}_2$ is almost homogeneous and inside field $\mathbf{E}_1$ can be analytically calculated as follows [6].

$$\mathbf{E}_1 = \frac{2\varepsilon_2}{\varepsilon_2 + \varepsilon_1} \mathbf{E}_2. \tag{2}$$

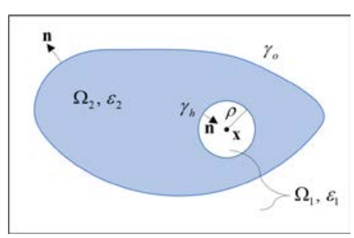

Fig. 1. Electrostatic system of two materials with a hole.

The hole sensitivity is defined as the variation of the objective function between before and after the hole generation when the hole is small enough.

$$S(\mathbf{x}) = \lim_{\rho \to 0} \frac{F(\mathbf{x},\rho) - F(\mathbf{x},0)}{A_h} = \lim_{\rho \to 0} \frac{\int_0^{\rho} \dot{F}(\mathbf{x},\rho) d\rho}{A_h}. \quad (3)$$

where, A_h is the area of the hole. Eq. (3) is rewritten using (1) and (2) [5].

$$S(\mathbf{x}) = 2\varepsilon_2 \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} \mathbf{E}(V) \cdot \mathbf{E}(\lambda). \quad (4)$$

III. Numerical Test

Two different methods are used to optimize a rotor shape of electrostatic actuator for generating the maximum torque. One is the conventional method with only the shape sensitivity analysis. The other includes the additional hole sensitivity information. Fig. 2 shows the numerical model driven by voltage switching in the clockwise direction. Dielectric is initially located in the design domain as the simplest topology. Since the energy difference between two switching positions should be maximized to generate the maximum torque, the objective function is as follows.

$$F = W_A - W_B \quad (5)$$

where, the subscripts A and B mean the switching positions. There is no necessity to solve the extra adjoint equation because the adjoint variable is the same to the state variable when the system energy is defined as the objective function. Therefore, the shape and hole sensitivities for the objective function are the function of the state variable only.

$$\dot{F} = \int_{\gamma} (\varepsilon_1 - \varepsilon_2) \left[\mathbf{E}_A(V_1) \cdot \mathbf{E}_A(V_2) - \mathbf{E}_B(V_1) \cdot \mathbf{E}_B(V_2) \right] V_n d\Gamma. \quad (6)$$

$$S(\mathbf{x}) = 2\varepsilon_2 \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} \left[\left| \mathbf{E}_A(V) \right|^2 - \left| \mathbf{E}_B(V) \right|^2 \right]. \quad (7)$$

In every iteration of the optimization process, the hole sensitivity is the largest at the selected position. When the hole sensitivity is large enough, the hole is generated. Then the shape sensitivity (6) on the whole material interfaces is calculated. The variation of the design variable is represented solving the level set equation below.

$$\frac{\partial \phi}{\partial t} + V_n \left| \nabla \phi \right| = 0 \quad (8)$$

where, ϕ is the level set function which the zero level is the material interface. The constraint of the constant area maintains the quantity of the dielectric during the optimization.

The shape and topology variations by two methods are shown in Fig. 3. The topologies of the optimum designs are totally different although the initial designs are the same. In Fig. 4, the variations of the objective function values by two methods are compared. The values of the objective function with two methods are almost similar in the earlier part of the optimization because the effective hole does not appear. The objective function values differ from each other when the topology of the model using the hole sensitivity method changes. Finally, the objective function value of the hole sensitivity case is larger about 25% than the conventional case.

Details of derivation for the hole sensitivity in electrostatic system and plenty examples will be presented in full paper.

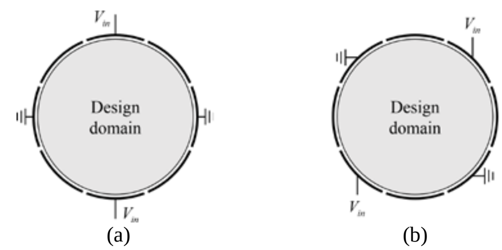

Fig. 2. Electrostatic actuator model. (a) Switching position A. (b) Switching position B.

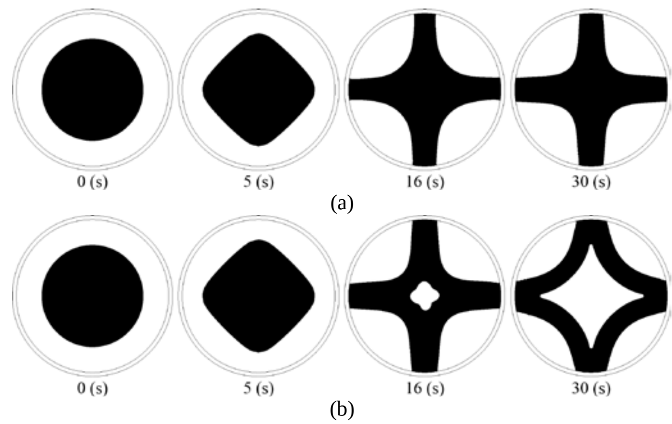

Fig. 3. Optimization process. (a) Conventional method. (b) Hole sensitivity method.

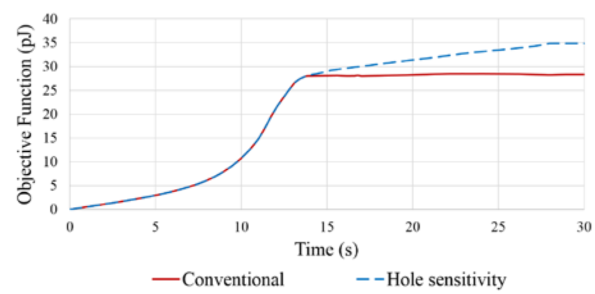

Fig. 4. Variation of objective function during optimization.

IV. References

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